Cosmic inflation

Successes of the hot Big Bang paradigm

Explains, *quantitatively*, several key observations (“four pillars"):

- cosmic expansion (Hubble’s law)
- cosmic microwave background
- light element abundances (BBN)
- formation of large-scale structure and galaxies

But basic hot Big Bang has some problems (other than unknown nature of dark matter and dark energy!).

Inflation, proposed by Alan Guth in 1981, is an “add on” very early (so it doesn’t spoil BBN) rapid phase of expansion that can solve some of these problems

Flatness problem

"The universe is nearly perfectly flat today, and was even flatter in the past"

Today: $|1 - \Omega_{\rm tot}(z=0)| < 0.01$ CMB (Planck) data

Why so close to flat? Physics equations allow for any value

Past:

$$H^2 = \frac{8\pi G \rho_{\rm tot}}{3} - \frac{kc^2}{a^2}$$

Friedmann eq.

$$\Rightarrow |1 - \Omega_{\rm tot}(t)| = \frac{|k|c^2}{H^2 a^2}$$

(midterm problem!)

Flatness problem (continued)

Assume radiation-dominated to simplify math
(most orders of magnitude in expansion from *a~0* to *a=1* occurred during radiation-dominated era, so if we can solve flatness problem for a radiation-dominated universe, we can solve it for our Universe)

$$a \propto t^{1/2} \Rightarrow \dot{a} \propto t^{-1/2}, \Rightarrow H \propto t^{-1}, \Rightarrow H^2 a^2 \propto t^{-1}$$

$$\Rightarrow |1 - \Omega_{\rm tot}(t)| \propto t$$

Small departure from flatness are amplified with time!

Flatness problem (continued)

Consider extremely conservative assumption (i.e., don't even rely on a precise measurement of present-day curvature)

$$|1 - \Omega_{\rm tot}(z = 0)| \lesssim 1$$

At decoupling (t~360,000 yr ~ 10^{13} s):

$$|1 - \Omega_{\rm tot}(t)| \lesssim \frac{t}{t_0} \sim \frac{10^{13}\ {\rm s}}{4 \times 10^{17}\ {\rm s}} \sim 10^{-5}$$

At matter-radiation eq. (t~47,000 yr ~ 10^{12} s): $|1 - \Omega_{\rm tot}(t)| \lesssim 10^{-6}$

At BBN (t~1 s): $|1 - \Omega_{\rm tot}(t)| \lesssim 10^{-18}$ ...

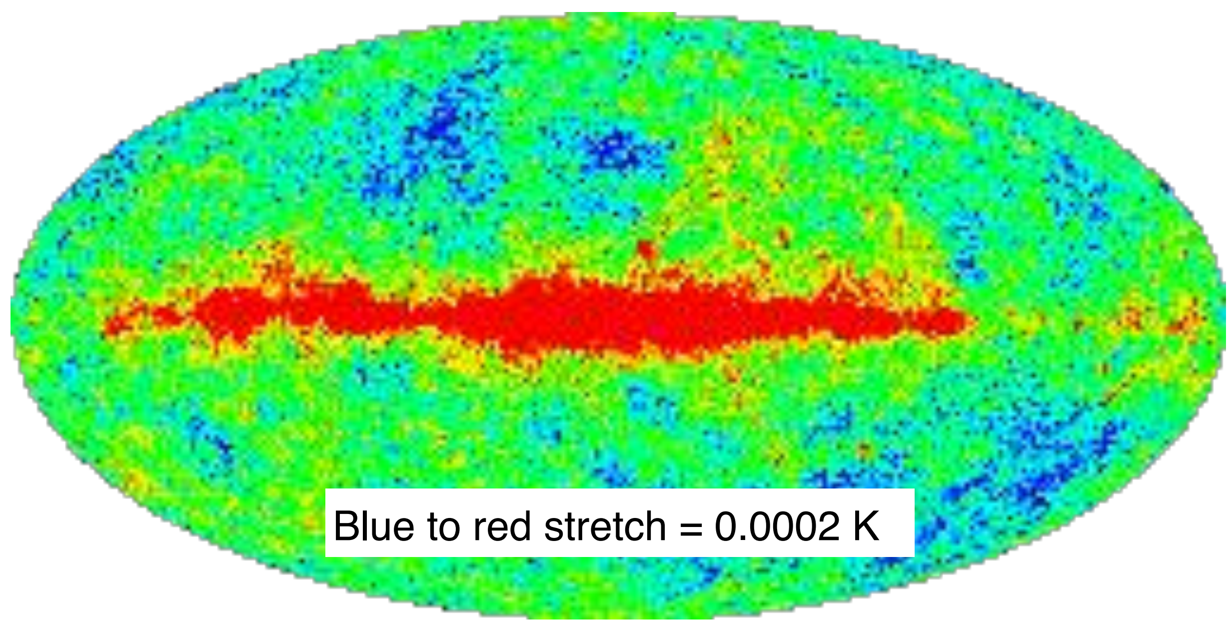
I.e., near flatness today is very likely not a coincidence, because it would require extreme fine tuning of conditions in the early universe.

Horizon problem

"How do regions that appear out of causal contact know about each other?"

Information cannot propagate faster than the speed of light. Sound waves in baryon-photon plasma propagate at $\sim c/3^{1/2}$, so the sound horizon decoupling ($\sim 1^{\circ} \sim 150$ comoving Mpc) is roughly the size of regions that were causally connected at last scattering.

How can regions separated by much larger angles on the sky be tuned to have the same T within $\Delta T/T \sim 10^{-5}$?



Superhorizon fluctuations exacerbate horizon problem

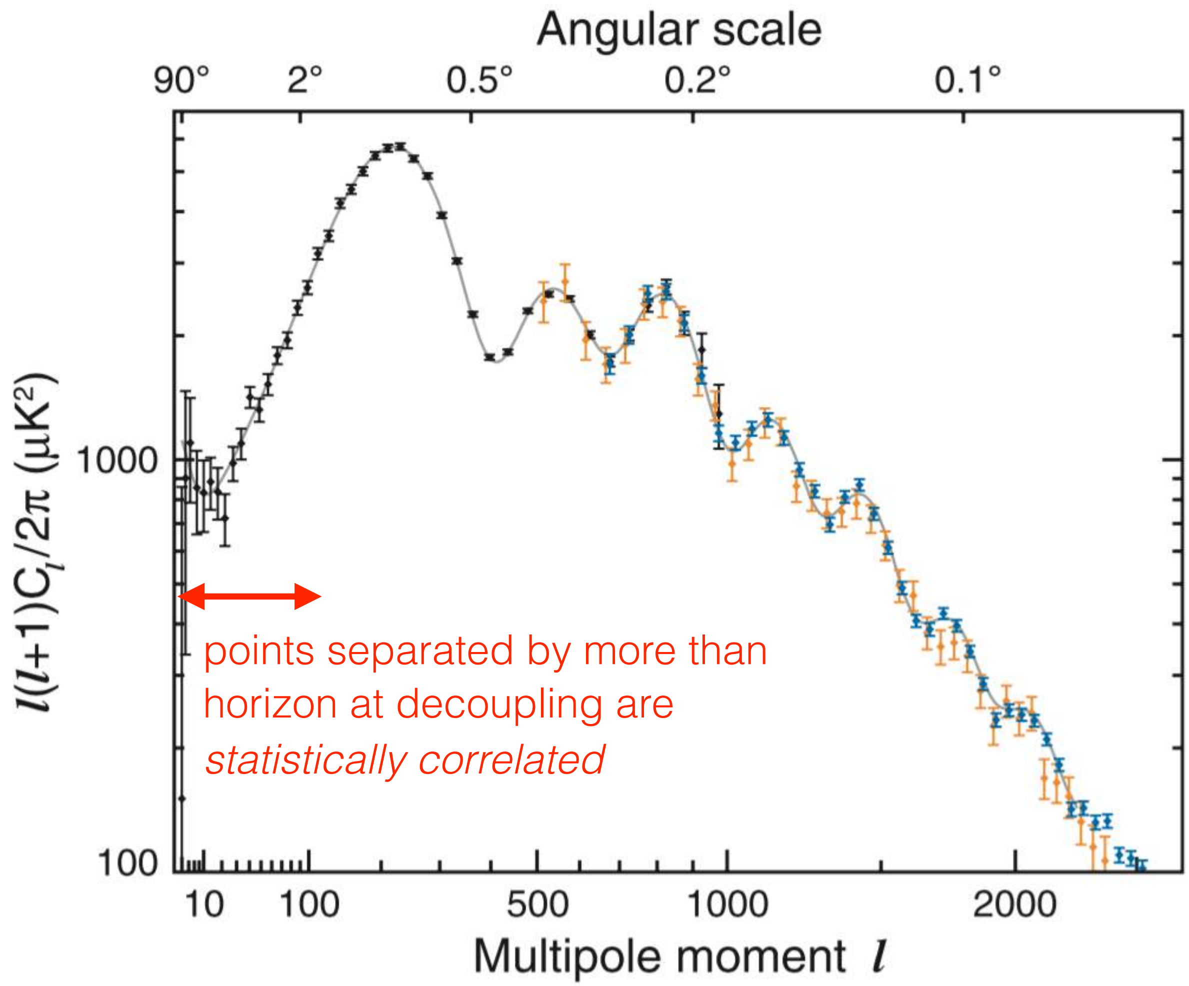


Figure 1. Compilation of the CMB data used in the nine-year *WMAP* analysis. The *WMAP* data are shown in black, the extended CMB data set—denoted "eCMB" throughout—includes SPT data in blue (Keisler et al. 2011) and ACT data in orange, (Das et al. 2011b). We also incorporate constraints from CMB lensing published by the SPT and ACT groups (not shown). The ΛCDM model fit to the *WMAP* data alone (shown in gray) successfully predicts the higher-resolution data.

(A color version of this figure is available in the online journal.)

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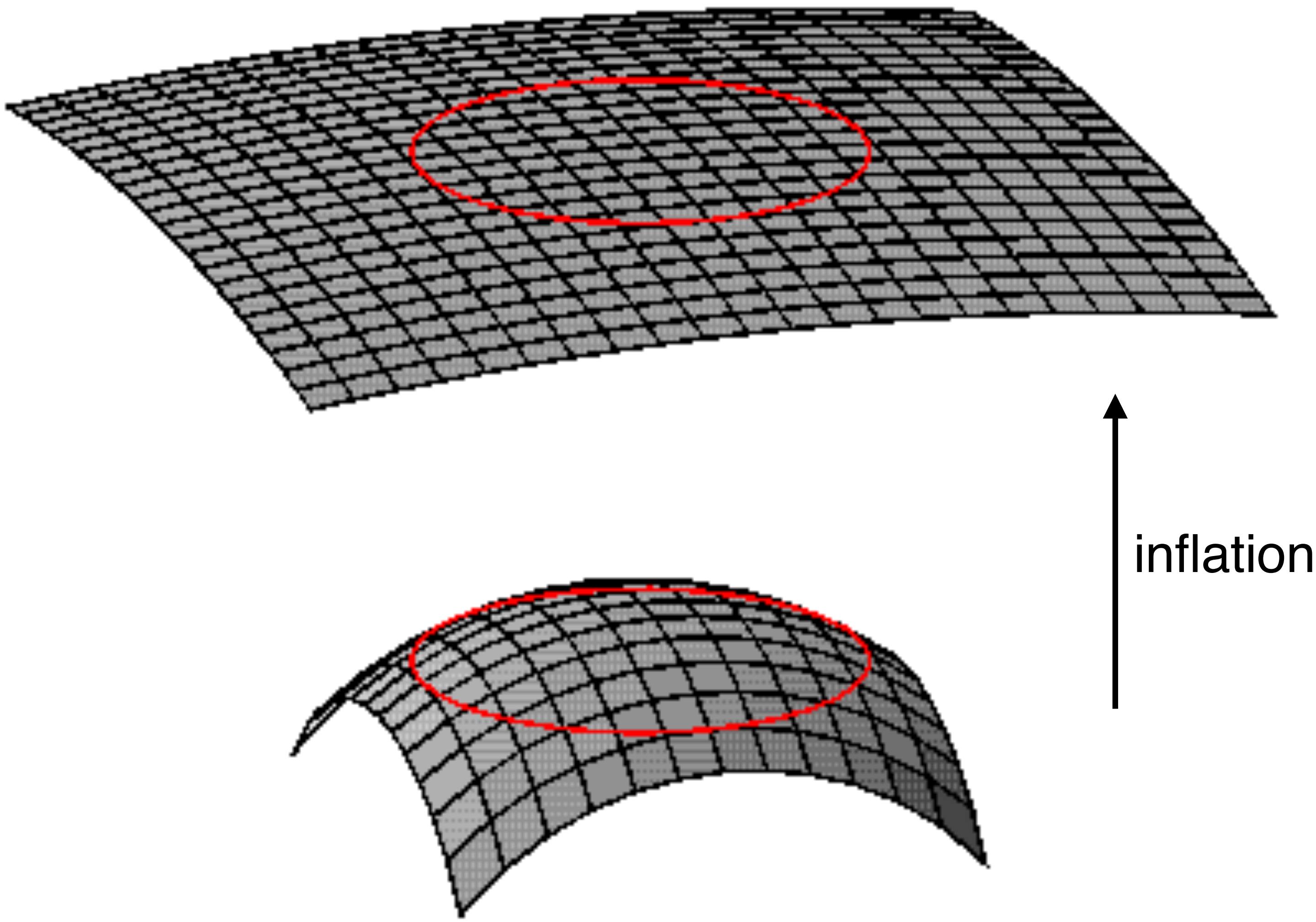
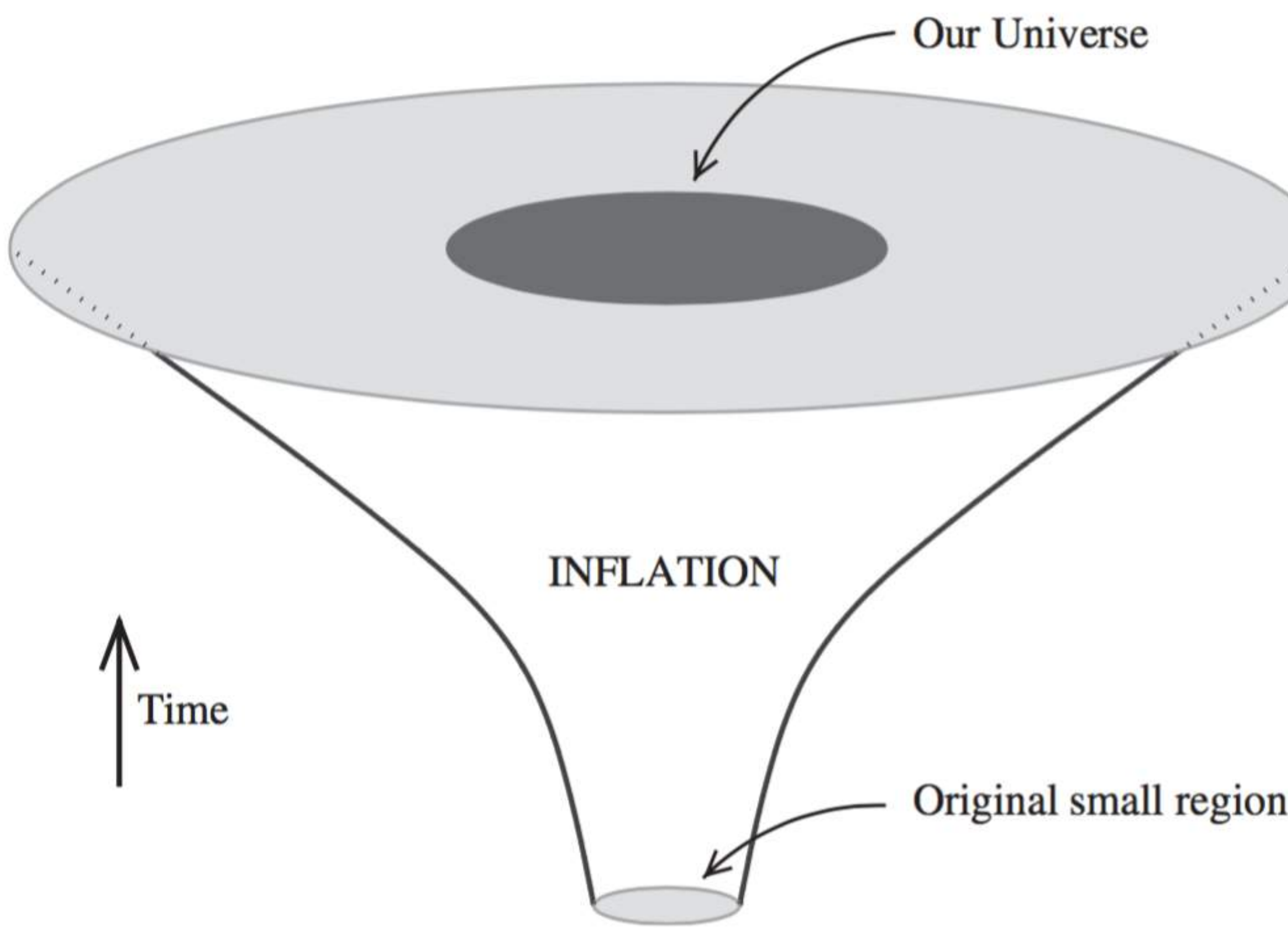
The inflationary solution to flatness problem: cartoon



During inflation, Universe expands much faster than the size of the observable universe (red circle). For observers, the Universe becomes indistinguishable from exactly flat.

Credit: E. Wright

The inflationary solution to horizon problem: cartoon



A small thermalized region initially causally connected is blown up to encompass a physical size larger than the entire observable universe. Today (and at last scattering) regions separated by this size no longer influence each other, but they did before inflation.

Liddle

The inflationary solution: math

Hypothetical very early phase of cosmic acceleration (similar to late-time dark energy):

$$\ddot{a} > 0 \iff \frac{d(\dot{a})}{dt} > 0 \iff \frac{d(Ha)}{dt} > 0$$

Then,

$$|1 - \Omega_{\rm tot}(t)| = \frac{|k|c^2}{H^2 a^2}$$

decreases with time, i.e. inflation will drive

$$\Omega_{\rm tot} \to 1$$

to arbitrarily high precision for sufficiently large and long-lasting acceleration.

Example: exponential inflation

Simplest form is analog of late-time cosmological constant

$$H^2 \approx \frac{\Lambda_{\rm infl}}{3} \Rightarrow a(t) \propto \exp\left(\sqrt{\frac{\Lambda_{\rm infl}}{3}}t\right)$$

Inflation must end (e.g., at BBN universe was radiation-dominated)

Assume end at $T_E \sim 10^{14}$ GeV, the “Grand Unification Theory” (GUT) scale. Motivation: in many particle physics-inspired models, inflation ends around that T.

Extrapolating back from $t \sim 13.7$ Gyr at $T \sim 3$ K, this corresponds to $t_E \sim 10^{-34}$ s.

Example: exponential inflation (continued)

To simplify math as before, assume expansion ~ radiation-dominated from end of inflation to today.

$$|1-\Omega_{\rm tot}(z=0)| \lesssim 0.01 \Rightarrow |1-\Omega_{\rm tot}(t_{\rm E})| \lesssim \frac{0.01\times 10^{-34}\ {\rm s}}{4\times 10^{17}\ {\rm s}}$$

$$\sim 2.5\times 10^{-54}$$

During inflation, *H*~const., so

$$|1-\Omega_{\rm tot}(t)| \propto \frac{1}{a^2(t)}$$

If $|1-\Omega_{\rm tot}| \sim 1$ before inflation, expansion by a factor ~10^{27} will flatten the universe by the required amount.

Example: exponential inflation (continued)

How many e-folds to solve the horizon problem?

Today's observable universe must fit within the comoving horizon at the beginning of inflation, i.e.

$$\frac{1}{a_0}\frac{c}{H_0} \lesssim \frac{1}{a_\mathrm{I}}\frac{c}{H_\mathrm{I}}$$

today   beginning of inflation

If $a \sim t^{1/2}$ between end of inflation and today,

$$\frac{a_0 H_0}{a_\mathrm{E} H_\mathrm{E}} \sim \frac{a_0}{a_\mathrm{E}}\left(\frac{a_\mathrm{E}}{a_0}\right)^2 \sim \frac{a_\mathrm{E}}{a_0} \sim \frac{T_0}{T_\mathrm{E}}$$

Example: exponential inflation (continued)

Numerically:
$$\frac{T_0}{T_{\rm E}} \sim \frac{10^{-3}\ \text{eV}}{10^{14}\ \text{GeV}} \sim 10^{-26}$$

$$\Rightarrow (a_{\rm I}H_{\rm I})^{-1} \gtrsim (a_0H_0)^{-1} \sim 10^{26}(a_{\rm E}H_{\rm E})^{-1}$$

I.e., $(aH)^{-1}$ must shrink by a factor ~10^{26} during inflation to solve the horizon problem.

Since H~const., a must increase by a factor ~10^{26} — about the same factor as required to solve the flatness problem

Minimum number of exponential e-folds: $N_{\rm min} \sim \ln\left(\frac{a_{\rm E}}{a_{\rm I}}\right) \sim 60$

Shortly after inflation was proposed, it was shown that it can also account for initial perturbations capable of growing into galaxies

THE DEVELOPMENT OF IRREGULARITIES IN A SINGLE BUBBLE INFLATIONARY UNIVERSE

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Received 25 June 1982

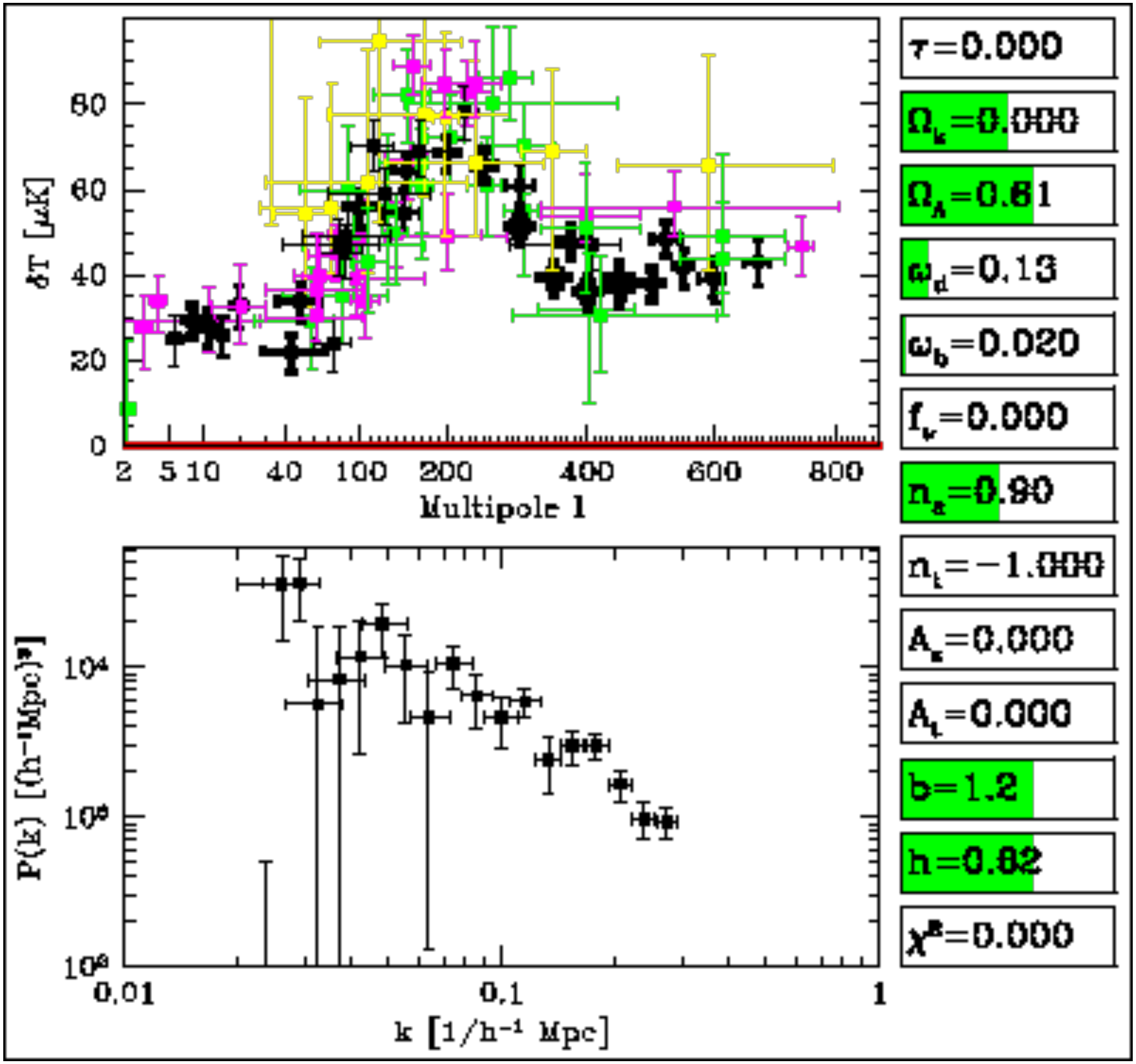
Quantum fluctuations on small scales get stretched to macroscopic scales

The horizon, flatness and monopole problems can be solved if the universe underwent an exponentially expanding stage which ended with a Higgs scalar field running slowly down an effective potential. In the downhill phase irregularities would develop in the scalar field. These would lead to fluctuations in the rate of expansion which would have the right spectrum to account for the existence of galaxies. However the amplitude would be too high to be consistent with observations of the isotropy of the microwave background unless the effective coupling constant of the Higgs scalar was very small.

and contemporaneous papers by

- Mukhanov & Chibisov (1981)
- Starobinsky (1982)
- Guth & Pi (1982)
- Bardeen, Steinhardt, & Turner (1983)

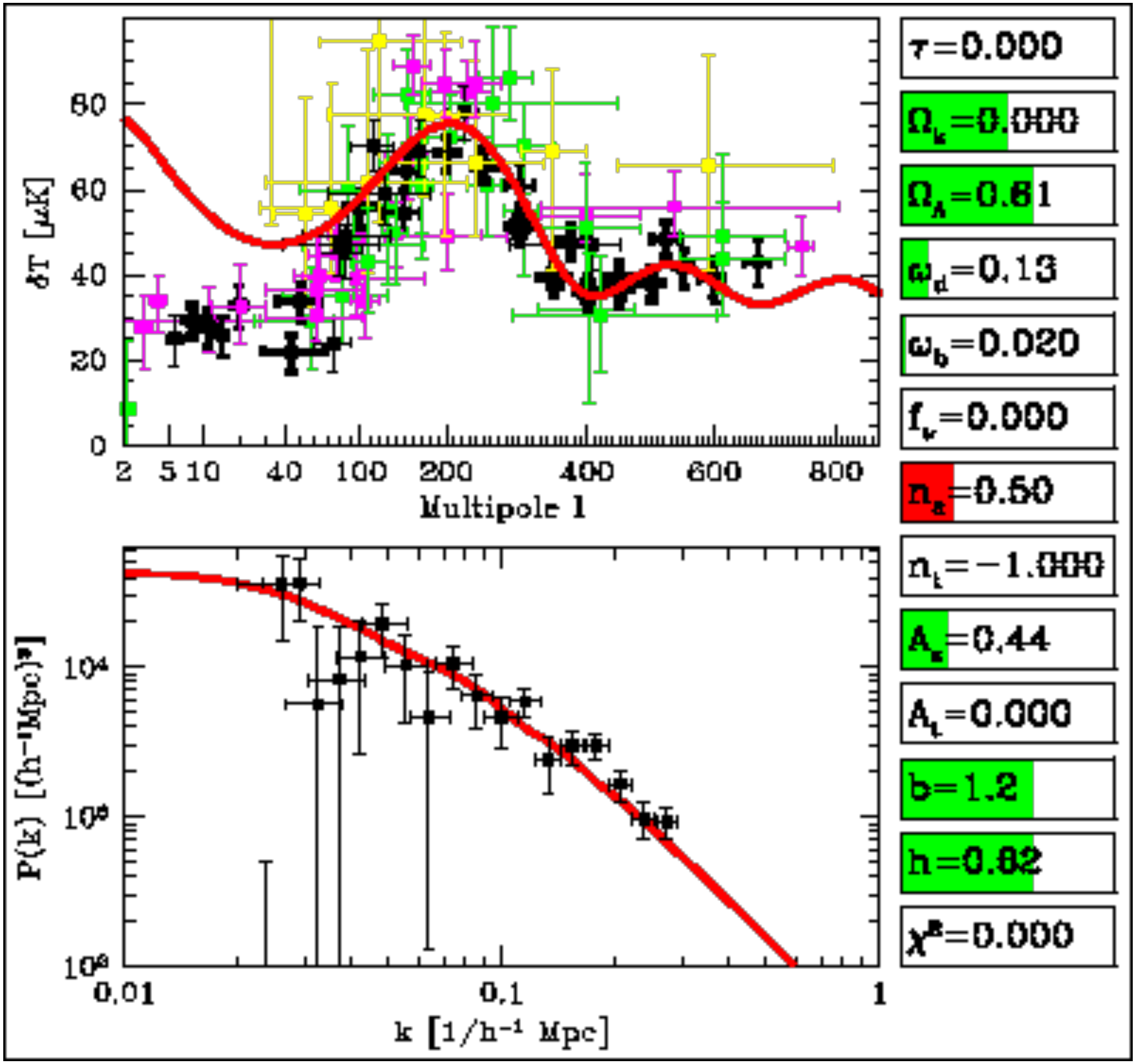
Varying amplitude of primordial power spectrum



$\omega_X \equiv \Omega_X h^2$

Credit: M. Tegmark

Varying spectral index of primordial power spectrum



$\omega_X \equiv \Omega_X h^2$

Credit: M. Tegmark

NINE-YEAR *WILKINSON MICROWAVE ANISOTROPY PROBE* (*WMAP*) OBSERVATIONS: COSMOLOGICAL PARAMETER RESULTS

ABSTRACT

We present cosmological parameter constraints based on the final nine-year *Wilkinson Microwave Anisotropy Probe* (*WMAP*) data, in conjunction with a number of additional cosmological data sets. The *WMAP* data alone, and in combination, continue to be remarkably well fit by a six-parameter ΛCDM model. When *WMAP* data are combined with measurements of the high-l cosmic microwave background anisotropy, the baryon acoustic oscillation scale, and the Hubble constant, the matter and energy densities, $\Omega_b h^2$, $\Omega_c h^2$, and Ω_Λ, are each determined to a precision of ∼1.5%. The amplitude of the primordial spectrum is measured to within 3%, and there is now evidence for a tilt in the primordial spectrum at the 5σ level, confirming the first detection of tilt based on the five-year *WMAP* data. At the end of the *WMAP* mission, the nine-year data decrease the allowable volume of the six-dimensional ΛCDM parameter space by a factor of 68,000 relative to pre-*WMAP* measurements. We investigate a number of data combinations and show that their ΛCDM parameter fits are consistent. New limits on deviations from the six-parameter model are presented, for example: the fractional contribution of tensor modes is limited to $r < 0.13$ (95% CL); the spatial curvature parameter is limited to $\Omega_k = -0.0027^{+0.0039}_{-0.0038}$; the summed mass of neutrinos is limited to $\sum m_\nu < 0.44$ eV (95% CL); and the number of relativistic species is found to lie within $N_{\rm eff} = 3.84 \pm 0.40$, when the full data are analyzed. The joint constraint on $N_{\rm eff}$ and the primordial helium abundance, $Y_{\rm He}$, agrees with the prediction of standard big bang nucleosynthesis. We compare recent *Planck* measurements of the Sunyaev–Zel'dovich effect with our seven-year measurements, and show their mutual agreement. Our analysis of the polarization pattern around temperature extrema is updated. This confirms a fundamental prediction of the standard cosmological model and provides a striking illustration of acoustic oscillations and adiabatic initial conditions in the early universe.

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The six parameters of the standard ΛCDM cosmological model

amplitude and slope of primordial power spectrum

Table 2
Maximum Likelihood ΛCDM Parameters[a]

Parameter	Symbol	*WMAP* Data	Combined Data[b]
	Fit ΛCDM Parameters		
Physical baryon density	$\Omega_b h^2$	0.02256	0.02240
Physical cold dark matter density	$\Omega_c h^2$	0.1142	0.1146
Dark energy density ($w = -1$)	Ω_Λ	0.7185	0.7181
Curvature perturbations, $k_0 = 0.002$ Mpc^{-1}	$10^9 \Delta^2_{\mathcal{R}}$	2.40	2.43
Scalar spectral index	n_s	0.9710	0.9646
Reionization optical depth	τ	0.0851	0.0800
	Derived Parameters		
Age of the universe (Gyr)	t_0	13.76	13.75
Hubble parameter, $H_0 = 100\,h$ km s^{-1} Mpc^{-1}	H_0	69.7	69.7
Density fluctuations @ 8 h^{-1} Mpc	σ_8	0.820	0.817
Baryon density/critical density	Ω_b	0.0464	0.0461
Cold dark matter density/critical density	Ω_c	0.235	0.236
Redshift of matter-radiation equality	z_{eq}	3273	3280
Redshift of reionization	z_{reion}	10.36	9.97

Notes.
[a] The maximum-likelihood ΛCDM parameters for use in simulations. Mean parameter values, with marginalized uncertainties, are reported in Table 4.
[b] "Combined data" refers to *WMAP*+eCMB+BAO+H_0.

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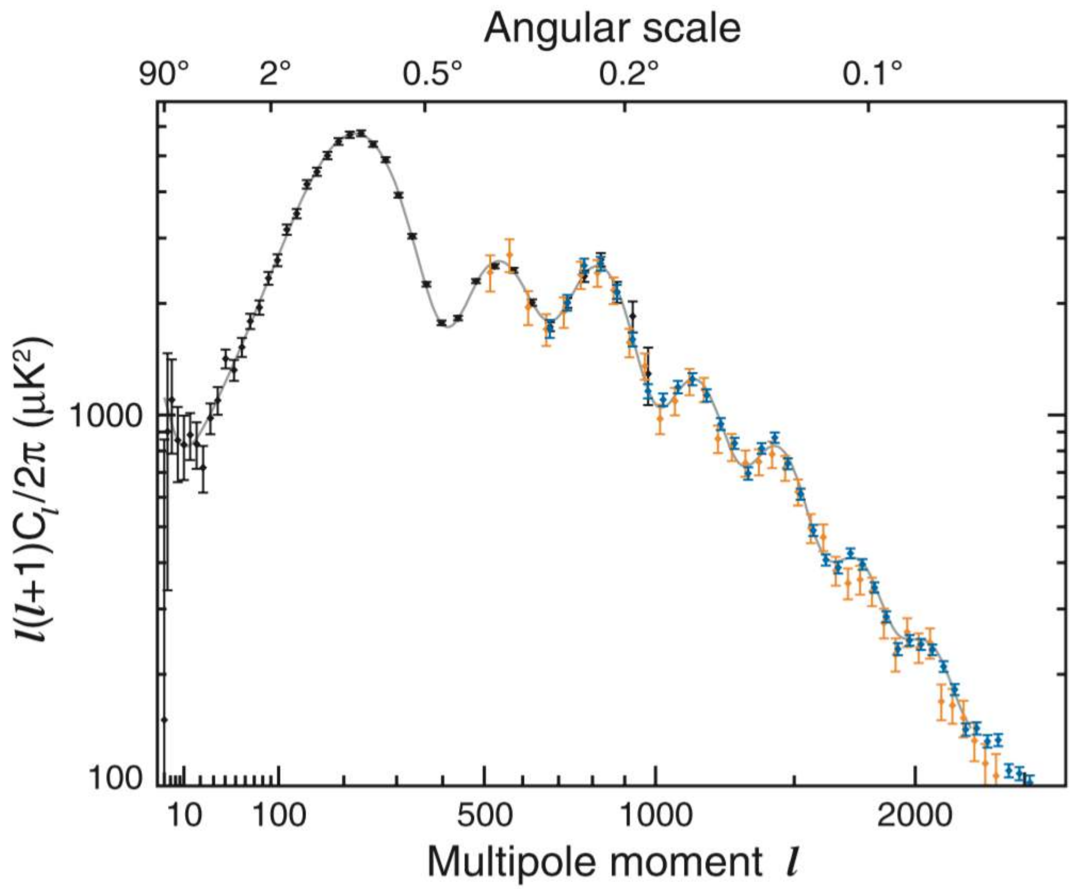
The six parameters of ΛCDM simultaneously fit a very large number of data points, e.g. CMB power spectrum



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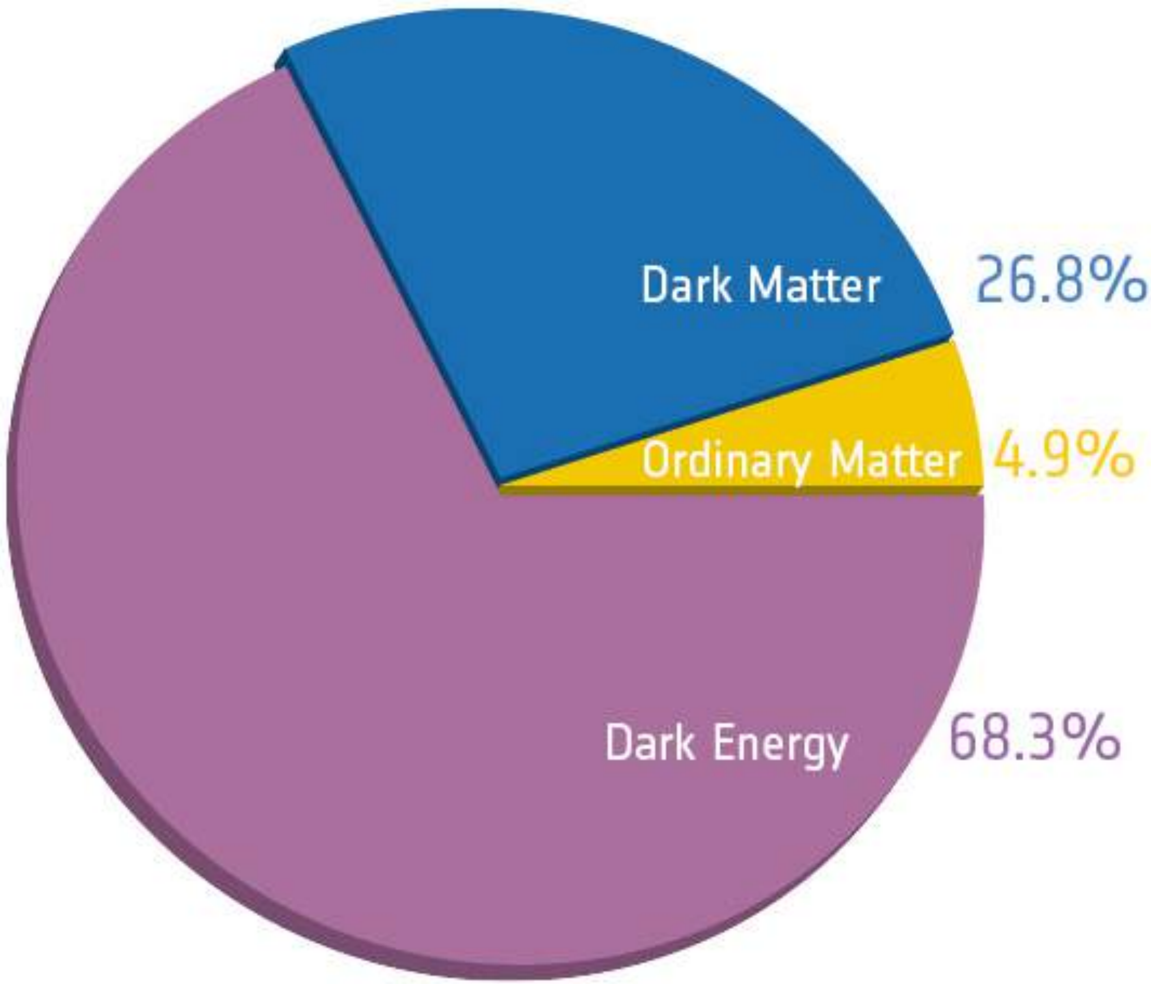
Conclusion: inflationary ΛCDM cosmology is an *excellent fit* but still *much physics to be understood*



Planck 2013

- Fits diverse observations (CMB, SNe, BAOs, light elements, gravitational lensing, galaxy rotation curves, growth of structure, …), with up to ~1% precision

- But all main elements (DE, DM, inflation) of unknown physical origin!